USN [ ][ ][ ][ ][ ][ ][ ][ ][ ][ ]   **06MAT31**

Third Semester B.E. Degree Examination, May/June 2010
Engineering Mathematics – III

Time: 3 hrs.   Max. Marks:100

Note: *Answer any FIVE full questions, selecting*
at least TWO questions from each part.

PART – A

1 a. Expand the function $f(x) = x - x^2$ in the interval $-\pi < x < \pi$. Deduce that
$$\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} \ldots\ldots$$ (07 Marks)

   b. Find the half-range cosine series for the function $f(x) = (x-1)^2$ in $0 < x < 1$.   (07 Marks)

   c. The following table gives the variations of periodic current over a period

t (sec) :	0	$T/6$	$T/3$	$T/2$	$2T/3$	$5T/6$	T
A (amp) :	1.98	1.30	1.05	1.30	-0.88	-0.25	1.98

   Show that there is a direct current part of 0.75 amp in the variable current and obtain the amplitude of the 1st harmonic.   (06 Marks)

2 a. Express the function $f(x) = \begin{cases} 1, & |x| < 1 \\ 0, & |x| > 1 \end{cases}$ as a Fourier integral and hence evaluate $\int_0^\infty \frac{\text{Sin}x}{x} dx$.
                                                                          (07 Marks)

   b. Find Fourier sine transform of $\frac{1}{x}e^{-ax}$.   (07 Marks)

   c. Use convolution theorem to find the inverse Fourier transform of $\frac{1}{(1+s^2)^2}$ given that $\frac{2}{1+s^2}$ is the Fourier transform of $e^{-|x|}$.   (06 Marks)

3 a. Form the partial differential equation by eliminating the arbitrary function from
$$Z = y^2 + 2f\left(\frac{1}{x} + \log y\right).$$ (07 Marks)

   b. Solve $\frac{\partial^2 z}{\partial x \partial y} = \text{Sin } x \text{ Sin } y$, given that $\frac{\partial z}{\partial x} = -2\text{ Sin}y$, when x = 0; and z = 0 when y is an odd multiple of $\pi/2$.   (07 Marks)

   c. Solve $x(y^2 - z^2) p + y(z^2 - x^2) q = z(x^2 - y^2)$.   (06 Marks)

4 a. Derive the one dimensional heat equation in the standard form.   (07 Marks)

   b. Obtain the various solutions of the Laplace's equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ by the method of separation of variables.   (07 Marks)

   c. A string stretched between the two fixed points (0, 0) and (1, 0) and released at rest from the position $y = \lambda \text{ Sin}(\pi x)$. Show that the formula for its subsequent displacement y (x, t) is $\lambda \text{ Cos}(c\pi t) \text{ Sin}(\pi x)$.   (06 Marks)

1 of 2

PART – B

5 a. Show that a real root of the equation tan x + tan hx = 0 lies between 2 and 3. Then apply the regula falsi method to find the third approximation. **(07 Marks)**

   b. Apply Gauss – Jordan method to solve the system of equations:
       $2x + 5y + 7z = 52; 2x + y - z = 0; x + y + z = 9.$ **(07 Marks)**

   c. Use power method to find the dominant eigen value and the corresponding eigen vector of the matrix $A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$ with the initial eigen vector as $[1, 1, 1]^T$. **(06 Marks)**

6 a. Under the suitable assumptions find the missing terms in the following table:

x :	-0.2	0.0	0.2	0.4	0.6	0.8	1.0
f (x) :	2.6	-	3.4	4.28	-	14.2	29

(07 Marks)

   b. Use Newton's divided difference formula to find f (4) given :

x :	0	2	3	6
f (x) :	-4	2	14	158

(07 Marks)

   c. Using Simpson's $\frac{3}{8}^{th}$ rule, evaluate $\int_{0}^{0.3} \sqrt{1-8x^3}\,dx$, by taking 7 ordinates. **(06 Marks)**

7 a. Solve the variational problem $\delta \int_{0}^{\pi/2} \left((y)^2 - (y')^2\right) dx$ under the conditions y (0) = 0, $y\left(\frac{\pi}{2}\right) = 2$. **(07 Marks)**

   b. Find the curve on which the function $\int_{0}^{\pi/2} \left[(y)^2 - (y')^2 - y \sin x\right]dx$ under the conditions $y (0) = y\left(\frac{\pi}{2}\right) = 0$, can be extremised. **(07 Marks)**

   c. Prove that the catenary is the plane curve which when rotated about a line (x – axis) generates a surface of revolution of minimum area. **(06 Marks)**

8 a. Find the Z – transform of i) n^2 ; ii) $n\,e^{-an}$. **(07 Marks)**

   b. Prove that : i) $Z (\cos n\theta) = \dfrac{z(z - \cos\theta)}{z^2 - 2z\cos\theta + 1}$; ii) $z (\sin n\theta) = \dfrac{z\sin\theta}{z^2 - 2z\cos\theta + 1}$. **(07 Marks)**

   c. Find the inverse Z – transform of $\dfrac{Z}{(Z-1)(Z-2)}$. **(06 Marks)**

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Third Semester B.E. Degree Examination, May/June 2010
Electronic Circuits

Time: 3 hrs.   Max. Marks:100

**Note: *Answer any FIVE full questions, selecting
at least TWO questions from each part.***

PART – A

1 a. Explain positive clipper and negative clipper, with necessary diagrams. **(08 Marks)**
  b. For the circuit shown in, Fig. Q1(b) :
     i) Sketch the output voltage waveform ii) What is the maximum positive output voltage?
     iii) What is the maximum negative output voltage? **(06 Marks)**
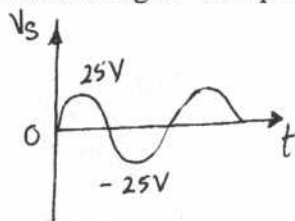
Fig. Q1(b)

  c. Explain SCHOTTKY DIODE construction and its application. **(06 Marks)**

2 a. Explain with a neat circuit diagram, voltage divider bias amplifier by mentioning the importance of bypass capacitor. **(06 Marks)**
  b. Obtain the graphical determination of AC emitter resistance of diode. **(06 Marks)**
  c. Draw the DC and AC equivalent circuits of voltage divider bias amplifier shown in Fig. Q2(c). **(08 Marks)**
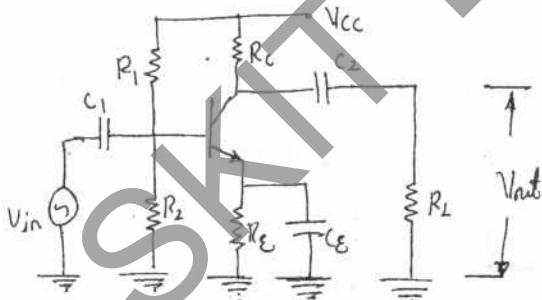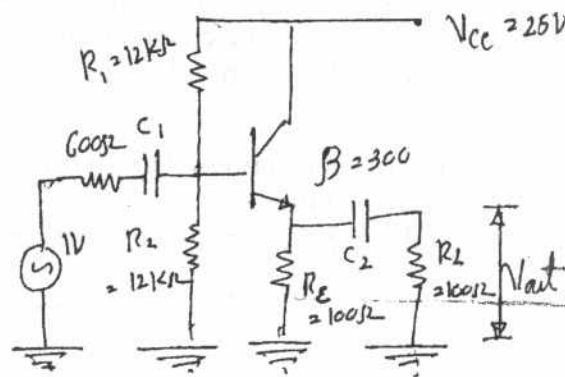

Fig. Q2(c)   Fig. Q3(c)

3 a. Obtain the expression for voltage gain of single stage CE voltage-divider bias amplifier using π-model. **(08 Marks)**
  b. Discuss trouble shooting of DC and AC circuits in voltage amplifier. **(06 Marks)**
  c. For the circuit shown in, Fig. Q3(c), calculate the voltage of output impedance. **(06 Marks)**

4 a. Explain power gain interms of voltage and current gain in power amplifier. **(06 Marks)**
  b. Show that the maximum efficiency of transformer coupled class A power amplifier is 50%. **(08 Marks)**
  c. In a class C power amplifier V_{CC} = 30 V; R_L = 10 k Ω current drain I_{dc} = 0.4 mA peak–to–peak output voltage $V_{out(p-p)}$ = 30 V. Calculate i) DC input power ii) AC input power iii) Efficiency. **(06 Marks)**

PART – B

5 a. Explain the principle of operation and structure of n-channel depletion mode MOSFET, with a neat sketch. **(06 Marks)**

   b. Discuss CMOS inverter with a neat circuit diagram, along with the transfer characteristics. **(06 Marks)**

   c. Obtain the equation for voltage gain of common-source D-MOSFET amplifier. **(08 Marks)**

6 a. Explain the frequency response of a typical AC amplifier, mentioning the importance of cut-off frequency. **(08 Marks)**

   b. Obtain the formula for decibel power gain and decibel voltage gain. **(04 Marks)**

   c. For the circuit shown in Fig. Q6(c). Calculate :
       i) The feedback fraction   ii) The ideal cloud leep voltage gain iii) The exact-cloud leep voltage gain iv) The percentage error between ideal and exact valves of the closed leep voltage gain. Assume the open leep voltage gain of op. amp as 10^5. **(08 Marks)**
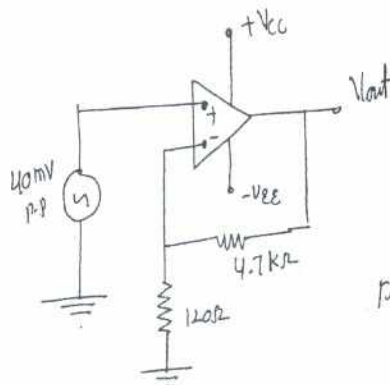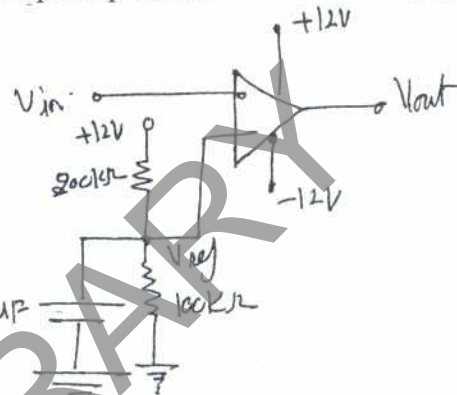

Fig. Q6(c)   Fig. Q7(b)

7 a. Explain the functional block diagram, of 555 timers. **(08 Marks)**

   b. The input voltage to the circuit shown in Fig. Q7(b) is a sine wave of peak value 8V.
       i) Calculate the trip point or threshold   ii) Calculate the cut-off frequency of the bypass circuit   iii) Sketch the output waveform and determine its duty circle. **(08 Marks)**

   c. Explain how Schmitt trigger can be used to convert a periodic sine wave to a rectangular wave. **(04 Marks)**

8 a. Explain the various characteristics on which a power supply depends, with respect to quality and suitability : **(04 Marks)**

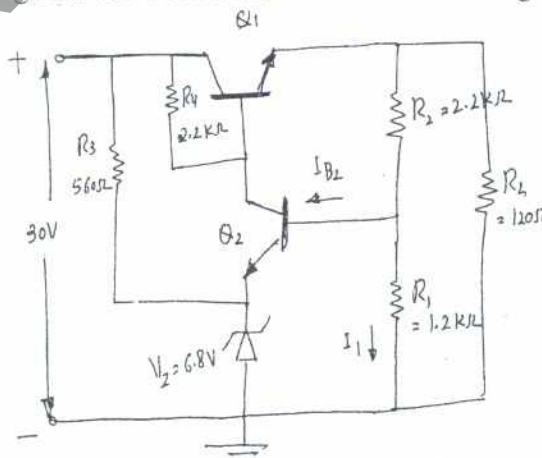   b. Calculate the output voltage for the circuit shown below in the Fig Q8(b) : **(08 Marks)**

Fig. Q8(b)

   c. Explain with a circuit diagram, unregulated DC to DC converter using power BJTs. **(08 Marks)**

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Third Semester B.E. Degree Examination, May/June 2010
Logic Design

Time: 3 hrs.   Max. Marks:100

Note: *Answer any FIVE full questions, selecting*
at least TWO questions from each part.

PART – A

1 a. Simplify the given Boolean function by using K-map method and express it in SOP form. Realise logic circuit by using NAND gates only.

$f(A, B, C, D) = \sum (m(7, 9, 10, 11, 12, 13, 14, 15))$ **(06 Marks)**

   b. Simplify following Boolean function by using K-map method in POS form:

$f(A, B, C, D) = \sum m(0, 1, 2, 3, 4, 5, 7)$ **(06 Marks)**

   c. Find prime implicants for the Boolean expression by using Quine McClusky method.

$f(A, B, C, D) = \sum (1, 3, 6, 7, 8, 9, 10, 12, 14, 15) + d(11, 13)$ **(08 Marks)**

2 a. Define decoder. Draw logic diagram of 3:8 decoder with enable input.   **(06 Marks)**
   b. Implement the given Boolean function by using 8:1 multiplexer.

$f(A, B, C, D) = \sum (0, 1, 3, 5, 7, 11, 12, 13, 14)$ **(06 Marks)**

   c. With a neat diagram, explain the decimal to BCD encoder.   **(08 Marks)**

3 a. What are the three different models for writing a module body in verilog HDL? Give example for any one model.   **(06 Marks)**
   b. With truth table and a neat logic diagram, explain full adder implementation. **(06 Marks)**
   c. Explain how IC 7483 can be used as 4 bit adder/subtractor.   **(08 Marks)**

4 a. With transfer characteristic, explain how Schmitt trigger converts a random waveform into a rectangular waveform.   **(06 Marks)**
   b. Explain basic S.R flip-flop by using NOR gate. What is the drawback of S-R flip-flop? How J-K flip-flop is obtained from S-R flip-flop?   **(08 Marks)**
   c. Find out characteristic equations of J-K flip-flop and D flip-flop.   **(06 Marks)**

PART – B

5 a. Explain any two types of shift register with waveforms. How Johnson counter is obtained from shift register?   **(10 Marks)**
   b. Design Mod-6 synchronous counter by using J-K flip-flop. Give excitation table of J-K flip-flop, state diagram and state transition table.   **(10 Marks)**

6 a. Differentiate between Moore and Mealy model of synchronous sequential circuit. **(04 Marks)**
   b. Reduce the state transition diagram of Mealy model by row elimination method and implication table method.   **(16 Marks)**
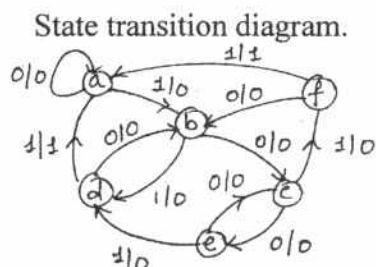
State transition diagram.

Fig. Q6 (b)

7 a. Explain with neat diagram, R-2R ladder type 4 bit D to A converter. Find out analog output if input is 1100 and '1'= +5 volts. For 10 bit DAC if full scale output is 10.24 volts, what is resolution? **(10 Marks)**
   b. Explain with a neat diagram, successive approximation type DAC. **(10 Marks)**

8 a. With a neat circuit diagram, explain the operation of a two input TTL NAND gate with tolem pole output. **(08 Marks)**
   b. Explain with a neat diagram, CMOS inverter. **(06 Marks)**
   c. Explain CMOS characteristics. **(06 Marks)**

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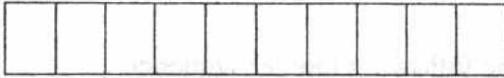
Third Semester B.E. Degree Examination, May/June 2010
Discrete Mathematical Structures

Time: 3 hrs.   Max. Marks:100

Note: *Answer any FIVE full questions, selecting*
at least TWO questions from each part.

PART – A

1 a. Define power set of a set. Find the power sets of the following set : $A = \{0, \phi, \{\phi\}\}$. **(04 Marks)**

 b. Using laws of set theory, prove that $(A \cap B) \cup \left[B \cap \left((C \cap D) \cup (C \cap \overline{D})\right)\right] = B \cap (A \cup C)$.

   (06 Marks)

 c. An integer is selected at random from 3 through 7 inclusive. If A is the event that a number divisible by 3 is chosen and B is the event that the number exceeds 10, determine $P_r(A)$, $P_r(B)$, $P_r(A \cap B)$ and $P_r(A \cup B)$ **(04 Marks)**

 d. A professor has two dozen textbooks on computer science and is concerned about their coverage of topics : (A) compilers, (B) data structures, and (C) operating systems. Following are the numbers of books that contain material on these topics : $|A| = 8$, $|B| = 13$, $|C| = 13$, $|A \cap B| = 5$, $|A \cap C| = 3$, $|B \cap C| = 6$, $|A \cap B \cap C| = 2$.
 i) How many of the textbooks include material on exactly one of these topics?
 ii) How many do not deal with any of the topics? **(06 Marks)**

2 a. Define the following : i) Proposition   ii) Tautology   iii) Contradiction.
   Determine whether the following compound statement is tautology or not :
   $$[(p \to q) \wedge (q \to r)] \to (p \to r)$$ **(08 Marks)**

 b. Using rules of inference, show that the following argument is valid :

   $$\begin{array}{l} p \\ p \to q \\ s \vee r \\ \underline{r \to \neg q} \\ \therefore s \vee t \end{array}$$ **(06 Marks)**

 c. Simplify the following switching network, (without using the truth table).   **(06 Marks)**

3 a. Establish the validity of the following argument:
   $$\begin{array}{l} \forall_x [p(x) \vee q(x)] \\ \exists_x \neg p(x) \\ \forall_x [\neg q(x) \vee r(x)] \\ \underline{\forall_x [s(x) \to \neg r(x)]} \\ \therefore \exists_x \neg s(x) \end{array}$$ **(10 Marks)**

 b. For the following statements the universe comprises all nonzero integers. Determine the truth value of each statement :
   i) $\exists_x \exists_y [xy = 1]$ ii) $\exists_x \forall_y [xy = 1]$ iii) $\forall_x \exists_y [xy = 1]$ **(06 Marks)**

 c. Negate and simplify each of the following : i) $\forall_x [p(x) \wedge \neg q(x)]$
   ii) $\exists_x [p(x) \vee q(x)] \to r(x)$ **(04 Marks)**

4 a. Define the following : i) Well-ordering principle
                         ii) Principle of mathematical induction.   **(04 Marks)**

 b. By the principle of mathematical induction, prove that :
   $$1^2 + 2^2 + 3^2 + \ldots\ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$$ **(06 Marks)**

4 c. Give a recursive definition for each of the following integer sequence :
       i) $c_n = 7n$ ii) $c_n = 2 - (-1)^n$. For $n \in z^+$.   **(04 Marks)**

    d. For $n \geq 0$ let F_n denote the nth Fibonacci number. Prove that for any positive integer n,

$$\sum_{i=1}^{n} \frac{F_{i-1}}{2^i} = 1 - \frac{F_{n+2}}{2^n}.$$

             (06 Marks)

PART – B

5 a. Define Cartesian product of two sets. For any three non-empty sets A, B, C. Prove that $A \times (B - C) = (A \times B) - (A \times C)$.   **(05 Marks)**

    b. Define the following : i) Function ; ii) Onto function ; iii) One – to – one. Let $f : z \rightarrow z$ be defined by $f(a) = a + 1$ for all $a \in z$. Find whether f is one-to-one correspondence or not.   **(05 Marks)**

    c. State the pigeonhole principle. Show that if any seven numbers from 1 to 12 are chosen, then two of them will add to 13.   **(05 Marks)**

    d. Let $f, g : R \rightarrow R$ be defined by $f(x) = 2x + 5$, $g(x) = \left(\frac{1}{2}\right)(x - 5)$. Show that f and g are invertible.   **(05 Marks)**

6 a. Let $A = \{1, 2, 3, 4\}$, Let $R = \{(1, 3), (1, 1), (3, 1), (1, 2), (3, 3), (4, 4)\}$ be the relation on A. Determine whether the relation R is reflexive, irreflexive, symmetric, antisymmetric or transitive.   **(05 Marks)**

    b. Let $A = \{1, 2)$, $B = \{m, n, p\}$ and $C = \{3, 4\}$. Let $R_1 = \{(1, m), \{(1, n), \{(1, p)\}$, $R_2 = \{(m, 3), (m, 4), (p, 4)\}$ and $R_3 = \{(m, 3), (m, 4), (p, 3)\}$. Prove that :
$$R_1 \circ (R_2 \cap R_3) \subseteq (R_1 \circ R_2) \cap (R_1 \circ R_3).$$
             (05 Marks)

    c. Let $A = \{a, b, c\}$, $B = P(A)$, where $P(A)$ is the power set of A. Let R is a subset relation on A. Draw the Hasse diagram of the poset (B, R).   **(05 Marks)**

    d. Let $A = \{2, 3, 4, 6, 8, 12, 24\}$ and let $\leq$ denotes the partial order of divisibility, that is $x \leq$ means x divides y. Let $B = \{4, 6, 12\}$. Determine :
       i) All upper bounds of B   ii) All lower bounds of B
       iii) Least upper bound of B   iv) Greatest lower bound of B.   **(05 Marks)**

7 a. For any group G prove that G is abelian if and only if $(ab)^2 = a^2 b^2$ for all a, b, $\in$ G. **(05 Marks)**

    b. State and prove Lagrange's theorem.   **(05 Marks)**

    c. A binary symmetric channel has probability $p = 0.05$ of incorrect transmission. If the word $c = 011011101$ is transmitted. What is the probability that :
       i) Single error occurs, ii) Three errors occur, no two of them consecutive?   **(05 Marks)**

    d. Determine the minimum distance between the code words, $E : Z_2^3 \rightarrow Z_2^6$

       $000 \rightarrow 000111$ $001 \rightarrow 001001$
       $010 \rightarrow 010010$ $011 \rightarrow 011100$
       $100 \rightarrow 100100$ $101 \rightarrow 101010$
       $110 \rightarrow 110001$ $111 \rightarrow 111000$

       How many errors can be detected and corrected by this code?   **(05 Marks)**

8 a. Construct a decoding table (with syndromes) for the group code given by the generator matrix: $G = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \end{bmatrix}$. Using this decoding table, decode the following received words :
       11110, 11011, 10000, 10101   **(10 Marks)**

    b. Determine whether $(z, \oplus, \odot)$ is a ring with the binary operations $x \oplus y = x + y - 7$, $x \odot y = x + y - 3xy$ for all x, y $\in$ z.   **(10 Marks)**

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Third Semester B.E. Degree Examination, May/June 2010
Data Structures with C

Time: 3 hrs.   Max. Marks:100

Note: *Answer any FIVE full questions, selecting at least TWO questions from each part.*

PART – A

1. a. What is a pointer? What are the uses of pointers in C? **(05 Marks)**

    b. Explain what is meant by lvalue and rvalue, with examples. **(05 Marks)**

    c. Write a C program to read ten integers and store them in an array using pointers. Print their sum and average. **(10 Marks)**

2. a. What is a string? How is a string declared and initialized? **(05 Marks)**

    b. Write appropriate structure definition and variable declarations to store following information about 100 students:
   Name, USN, Gender, Date of birth and marks in three subjects S_1, S_2 & S_3.
   Date of birth should be a structure containing fields day, month and year. **(05 Marks)**

    c. Write a function that given a binary file, copies the odd items (item 1,3,5,.....n) to a second binary file and the even items(item 2,4,6,8,...n+1) to a third binary file. **(10 Marks)**

3. a. Define stack. Briefly explain the primitive operations on the stack. **(05 Marks)**

    b. Show using the tabular columns, how the expression (A+B)*C is converted into a postfix expression according to the infix to postfix conversion algorithm. **(05 Marks)**

    c. Write the algorithm to evaluate a valid postfix expression and hence evaluate the postfix expression :
         6   2   3   +   –   3   8   2   /   +   *
   All the operands are single digit positive integers and operators are binary in nature. **(10 Marks)**

4. a. Determine what the following recursive C function computes:

```
int func(int n)
{
    if (n == 0)
        return(0);
    return(n + func(n – 1));
} /* end of func */
```
   Write an iterative function to accomplish the same. **(05 Marks)**

    b. Explain the working of a simple queue. **(05 Marks)**

    c. Write a recursive function fact(n) to find the factorial of an integer. Diagrammatically explain, how the stacking and unstacking takes place during execution for fact(4). **(10 Marks)**

PART – B

5   a.   What is a linear linked list? Write the algorithm to add an element to the front of the list.
(05 Marks)

    b.   What are the advantages and disadvantages of representing a group of items as an array versus linear linked list? (05 Marks)

    c.   Write the following C routines for the dynamic implementation of a linked list. NODEPTR is of type pointer to a node.
        i)   void insertafter(NODEPTR p, int x)   which inserts a node with information x after a node pointed to by p.
        ii) void place(NODEPTR *plist, int x)   which inserts a node with information x at a proper position within the linear linked list pointed to by *plist. The list is assumed to contain information in the increasing order. (10 Marks)

6   a.   What is a circular list? Explain with a diagram. (05 Marks)

    b.   Compare linear linked list and doubly linked list, with diagrams. (05 Marks)

    c.   Give the C implementation of stack as circular list. (10 Marks)

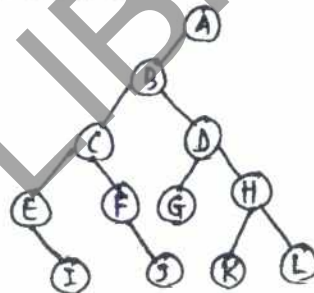7   a.   With reference to the b-tree in Fig.Q7(a), give the three traversals (05 Marks)

Fig.Q7(a)

    b.   i)   Define strictly binary tree. Is the tree in Fig.Q7(a), a strictly b-tree.
        ii) Define almost-complete b-tree. Is the tree in Fig.Q7(a), an almost complete b-tree. (05 Marks)

    c.   With reference to the dynamic node representation of b-tree, write the following C routines:
        i)   NODEPTR maketree(int x) which creates a node with information x.
        ii) Void setleft(NODEPTR, int x) which sets a node with contents x as the left son of the node pointed to by p. (10 Marks)

8   a.   With an example, show how a list can be represented as binary tree. (05 Marks)

    b.   Define the following terms with reference to general trees:
        Father, son, brother, forest and ordered tree. (05 Marks)

    c.   Give the node structure of an expression tree. Explain how the expression is evaluated. (10 Marks)

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Third Semester B.E. Degree Examination, May/June 2010
UNIX and Shell Programming

Time: 3 hrs.   Max. Marks:100

Note: *Answer any FIVE full questions, selecting*
at least TWO questions from each part.

PART – A

1 a. Explain salient features of UNIX operating system.   (06 Marks)
   b. Explain the different types of files supported in UNIX.   (06 Marks)
   c. Explain the following commands, with example :
       mailx, passwd, stty, who   (08 Marks)

2 a. Briefly describe the different ways of setting file permissions.   (06 Marks)
   b. What are the three modes of vi editor? Explain.   (06 Marks)
   c. What is a navigation? What are commands used for navigation in vi editor? (08 Marks)

3 a. What are environmental variables? State their significance.   (06 Marks)
   b. What are three standard files used by UNIX commands? Explain.   (06 Marks)
   c. What is shell process? What are three different phases in the creation of process? (08 Marks)

4 a. What are hard links and symbolic links?   (06 Marks)
   b. Explain with an example, find command and its operators.   (06 Marks)
   c. Explain the following filters, with examples :
       head, tail, cut, tr.   (08 Marks)

PART – B

5 a. How to search for a pattern using grep? What are the options used by grep? (08 Marks)
   b. Explain extended regular expression (ERE) set used by grep.   (06 Marks)
   c. What are internal commands used by sed?   (06 Marks)

6 a. What are the special parameters used by the shell?   (06 Marks)
   b. Explain how numeric and string comparison is done by using test.   (06 Marks)
   c. Write a menu driven shell script to display list of files, process of user, todays date and users
       of the system.   (08 Marks)

7 a. Explain any three built in variables used in awk.   (06 Marks)
   b. Give the syntax of three control flow statements used by awk.   (06 Marks)
   c. Explain built in functions used in awk.   (08 Marks)

8 a. Explain string handling function in Perl.   (06 Marks)
   b. Explain split and join functions.   (06 Marks)
   c. Write a Perl script to convert a decimal number to binary.   (08 Marks)

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Third Semester B.E. Degree Examination, May/June 2010
Advanced Mathematics – I

Time: 3 hrs.   Max. Marks:100

Note: *Answer any FIVE full questions.*

1   a.   Express the complex number $\dfrac{(1+i)(1+3i)}{1+5i}$ in the form $x + iy$.   (06 Marks)

    b.   Prove that $(1+i)^n + (1-i)^n = 2^{\frac{n}{2}+1}\cos\left(\dfrac{n\pi}{4}\right)$.   (07 Marks)

    c.   Expand $\cos^8\theta$ in a series of cosines multiples of θ.   (07 Marks)

2   a.   Find the nth derivative of $e^{ax}\sin(bx+c)$.   (06 Marks)

    b.   If $y = a\cos(\log x) + b\sin(\log x)$, prove that $x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$.   (07 Marks)

    c.   Find the nth derivative of $\dfrac{x}{(x-1)(2x+3)}$.   (07 Marks)

3   a.   State Taylor's theorem and expand the polynomial $2x^3 + 7x^2 + x - 6$ in powers of $(x-1)$.   (06 Marks)

    b.   Expand $\tan x$ in ascending powers of x using MacLaurin's theorem upto the term containing x^4.   (07 Marks)

    c.   If $Z = \dfrac{x^2+y^2}{x+y}$ prove that $\left(\dfrac{\partial z}{\partial x} - \dfrac{\partial z}{\partial y}\right)^2 = 4\left(1 - \dfrac{\partial z}{\partial x} - \dfrac{\partial z}{\partial y}\right)$.   (07 Marks)

4   a.   If $u = \tan^{-1}\left(\dfrac{x^3+y^3}{x-y}\right)$, prove that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = \sin 2u$.   (06 Marks)

    b.   If $u = f(x, y)$ where $x = r\cos\theta$ and $y = r\sin\theta$, prove that $\left(\dfrac{\partial u}{\partial x}\right)^2 + \left(\dfrac{\partial u}{\partial y}\right)^2 = \left(\dfrac{\partial u}{\partial r}\right)^2 + \dfrac{1}{r^2}\left(\dfrac{\partial u}{\partial \theta}\right)^2$.   (07 Marks)

    c.   If $u = x^2 - 2y$, $v = x + y + z$ and $w = x - 2y + 3z$, find the value of $J\left(\dfrac{u, v, w}{x, y, z}\right)$.   (07 Marks)

5   a.   Obtain the reduction formula for $\int \sin^m x \cos^n x \, dx$.   (06 Marks)

    b.   Evaluate $\displaystyle\int_0^a \dfrac{x^7}{\sqrt{a^2-x^2}} \, dx$.   (07 Marks)

    c.   Evaluate $\displaystyle\int_0^1\int_0^x e^{\left(\frac{y}{x}\right)} dy \, dx$.   (07 Marks)

6 a. Evaluate $\int\limits_{0}^{1}\int\limits_{0}^{\sqrt{1-x^2}}\int\limits_{0}^{\sqrt{1-x^2-y^2}}xyz\,dz\,dy\,dx$. (06 Marks)

   b. Prove that $\beta(m,n)=\dfrac{\overline{|m}\,\overline{|n}}{\overline{|m+n}}$. (07 Marks)

   c. Show that $\int\limits_{0}^{\pi/2}\sqrt{\sin\theta}\,d\theta\times\int\limits_{0}^{\pi/2}\dfrac{d\theta}{\sqrt{\sin\theta}}=\pi$. (07 Marks)

7 a. Solve $3\,e^{x}\tan y\,dx+(1-e^{x})\sec^{2}y\,dy=0$. (06 Marks)
   b. Solve $x^{2}y\,dx=(x^{3}+y^{3})\,dy$. (07 Marks)
   c. Solve $x\dfrac{dy}{dx}+y=x^{3}y^{6}$. (07 Marks)

8 a. Solve $\dfrac{d^{2}y}{dx^{2}}+\dfrac{dy}{dx}=x^{2}+2x+4$. (06 Marks)

   b. Solve $\dfrac{d^{2}y}{dx^{2}}+3\dfrac{dy}{dx}+2y=4\cos^{2}x$. (07 Marks)

   c. Solve $\dfrac{d^{3}y}{dx^{3}}+2\dfrac{d^{2}y}{dx^{2}}+\dfrac{dy}{dx}=e^{-x}+\sin 2x$. (07 Marks)

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